Metrologia **42** (2005) 71–80 doi:10.1088/0026-1394/42/2/001

Redefinition of the kilogram: a decision whose time has come

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Received 23 December 2004
Published 28 February 2005
Online at stacks.iop.org/Met/42/71

Abstract
The kilogram, the base unit of mass in the International System of Units (SI), is defined as the mass $m(\mathcal{K})$ of the international prototype of the kilogram. Clearly, this definition has the effect of fixing the value of $m(\mathcal{K})$ to be one kilogram exactly. In this paper, we review the benefits that would accrue if the kilogram were redefined so as to fix the value of either the Planck constant h or the Avogadro constant N_A instead of $m(\mathcal{K})$, without waiting for the experiments to determine h or N_A currently underway to reach their desired relative standard uncertainty of about 10^{-8}. A significant reduction in the uncertainties of the SI values of many other fundamental constants would result from either of these new definitions, at the expense of making the mass $m(\mathcal{K})$ of the international prototype a quantity whose value would have to be determined by experiment. However, by assigning a conventional value to $m(\mathcal{K})$, the present highly precise worldwide uniformity of mass standards could still be retained. The advantages of redefining the kilogram immediately outweigh any apparent disadvantages, and we review the alternative forms that a new definition might take.

1. Introduction

Of the seven base units of the International System of Units (the SI)—the metre, kilogram, second, ampere, kelvin, mole and candela—only the kilogram is still defined in terms of a material artefact. Its definition reads ‘The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram’ [1]. Nevertheless, because of the way they are defined, three other base units of the SI call upon the definition of the kilogram, namely the ampere, the mole and the candela. Thus, any uncertainty inherent in the definition of the kilogram propagates also into these units.

The international prototype (normally indicated by the symbol $\mathcal{K}$), a cylinder with a height and diameter of about 39 mm, is made of an alloy of platinum and iridium with mass fractions of 90 % and 10 %, respectively [2]. The mass of the international prototype was designated the unit of mass in the metric system in 1889 by the 1st General Conference on Weights and Measures (CGPM), and has continued to play that role in the SI, which was established by the 11th CGPM in 1960 [1]. Together with its six official copies, the international prototype is kept in a vault at the International Bureau of Weights and Measures (BIPM) at Sèvres, on the outskirts of Paris.

Although the international prototype has served science and technology well as a standard of mass during the last 115 years, as a material artefact it has one important limitation: it is not linked to an invariant of nature. Thus, it can be damaged or even destroyed, it collects dirt from the ambient atmosphere and must be carefully washed in a prescribed way prior to use, it cannot be used routinely for fear of wear, and it seems that its mass may be changing with time with respect to the ensemble of Pt–Ir standards of about the same age—perhaps 50 µg per century (or possibly significantly more), corresponding to a fractional change of 5×10^{-8} per 100 years [2–4]. And of course, it can be accessed only at the BIPM. Most important, notwithstanding the present worldwide consistency of Pt–Ir mass standards of 1 kg, which is a few times 10^{-9} kg (a few micrograms), and our present ability to compare such standards with an uncertainty even smaller than this, the drift of the worldwide ensemble of one-kilogram Pt–Ir standards relative to an invariant of nature is unknown at a level below 1 mg over a period of 100 or even 50 years [2].

Because of these difficulties, an international effort has been underway for over 25 years to relate the mass $m(\mathcal{K})$ of the international prototype to a fundamental constant, or to the mass of an atom or a fundamental particle, with an uncertainty that is sufficiently small to allow the current definition of the kilogram to be replaced. A relative standard uncertainty u_r (estimated standard deviation) of about 10^{-8} in relating $m(\mathcal{K})$ to a fundamental constant or atomic mass has been generally regarded as being a desirable goal to achieve before the definition of the kilogram should be revised [2, 3]. However, the two experimental approaches that are most advanced would relate $m(\mathcal{K})$ to either the Planck constant h or the Avogadro constant N_A, and neither of these has yet reached a relative uncertainty of much less than 10^{-7}. Indeed, at the present time there is a difference of nearly 1 part in 10^6 between the results of the two approaches [5].

It is the purpose of this paper to demonstrate that there is actually no need to wait for the experiments to improve. If the changeover were to be made now to a new definition that fixes either h or N_A, the uncertainties of the SI values of many fundamental constants would be immediately reduced by more than a factor of ten, with significant advantages to our practical measurement systems, especially those that deal with the measurement of electrical quantities. The price to be paid would be that the mass of the international prototype $m(\mathcal{K})$ would no longer be known exactly, but would have to be determined by experiment. However, by adopting a conventional value for $m(\mathcal{K})$ the present worldwide system of mass metrology would not be significantly affected, nor would the three other units of the SI that are dependent upon the kilogram. Therefore, there is everything to be gained by redefining the kilogram immediately without waiting for the anticipated experimental advances.

2. The watt balance and the x-ray crystal density experiments

The two experimental approaches opening the way to a new definition that are most advanced are the moving-coil watt balance [6, 7] and the x-ray crystal density (XRCD) method using silicon [8, 9].

The watt balance allows one to determine a virtual power mechanically in terms of length, mass and time, as well as electrically in terms of voltage and resistance based on the Josephson effect and quantum Hall effect, respectively. The result is an experimental determination of the Planck constant if one accepts the present definition of the kilogram, or an experimental determination of the mass of an unknown standard of mass if one takes the Planck constant h to be a known quantity. This leads naturally to the idea of redefining the kilogram so as to fix the value of h, and then using the watt balance to realize the definition, although such a definition could be realized by any physical experiment linking electrical to mechanical quantities that could be carried out with the required accuracy.

In the silicon XRCD method, one measures the lattice spacing d_{220} of a very pure, nearly crystallographically perfect single crystal of silicon, its macroscopic mass density and the mean molar mass of the silicon atoms of which it is composed (the latter by determining the mole fractions of the three naturally occurring silicon isotopes in the crystal). In this case, the result is an experimental determination of the Avogadro constant if one accepts the present definition of the kilogram, or an experimental determination of the mass of the crystal if one takes the Avogadro constant N_A to be a known quantity. This naturally leads to the idea of redefining the kilogram so as to fix the value of N_A, and then using the XRCD method to realize the definition. However, as with the previous definition based on a fixed value for h, realization of a definition based on a fixed value for N_A would not be limited to the XRCD method, but would be open to any physical experiment that could count microscopic entities with sufficient accuracy.

It is important to recognize that no matter which of the two definitions is chosen, the method of realizing it is not tied to the definition. In particular h and N_A are related through the fine-structure constant α and other well-known constants by equation (B6) of appendix B, so that any experiment that may be used to determine either of these constants could be used to realize the kilogram for either the fixed-h or the fixed-N_A definition. In appendix A we suggest possible wordings for new definitions of the kilogram that fix the value of either h or N_A, and we review the merits of each of the two different types of definitions.

Although u_r of the watt balance and silicon XRCD experiments are both, still, one to two orders of magnitude larger than the value $u_r \approx 10^{-8}$ generally considered desirable prior to proceeding with a redefinition of the kilogram, we present here the arguments for proceeding with such a redefinition without delay. If this were done, the international prototype would be retained as a working, 'conventional' reference standard of mass. In this way, the present excellent worldwide uniformity of one-kilogram Pt–Ir mass standards would be maintained, while at the same time the many benefits of having either h or N_A exactly known would be realized. Moreover, each SI base unit would then be defined in terms of invariants. We show how all of this might be achieved in what follows, and begin by first reviewing, based on the best data currently available, (i) the impact that a redefinition that fixes either h or N_A would have on the uncertainties of the values of many fundamental constants, and on the results of various electrical measurements; and (ii) how well we would know the mass of the international prototype in terms of the mass unit defined by either of the new definitions.

3. Impact of new definitions on the values of the constants

For simplicity, the details of how one obtains best values of the fundamental constants when the kilogram is defined so as to fix the value of either the Planck constant h or the Avogadro constant N_A are given in appendix B of this paper. Suffice it to say here that one uses the data and procedures employed in the 2002 Committee on Data for Science and

Table 1. Relative standard uncertainties u_r of a representative group of fundamental constants whose values depend on the mass $m(\mathcal{K})$ of the international prototype, as determined by the 2002 CODATA final adjustment, for three different definitions of the kilogram.

Constant[a]	$m(\mathcal{K})$ fixed (CODATA 2002) $10^8 u_r$	h fixed $10^8 u_r$	N_A fixed $10^8 u_r$
$m(\mathcal{K})$	0	17	17
h	17	0	0.67
N_A	17	0.67	0
m_e	17	0.67	0.044
m_p	17	0.67	0.013
e	8.5	0.17	0.50
K_J, Φ_0	8.5	0.17	0.17
γ_p	8.6	1.3	1.1
F	8.6	0.83	0.50
μ_B	8.6	0.83	1.2
μ_N	8.6	0.83	1.2
V_{90}/V	8.5	0.17	0.17
A_{90}/A	8.5	0.17	0.50
W_{90}/W	17	0	0.67
u, m_u	17	0.67	0
c_1, c_{1L}	17	0	0.67
J in eV	8.5	0.17	0.50
kg in u	17	0.67	0
m^{-1} in kg	17	0	0.67

[a] Here m_e is the electron mass, m_p the proton mass, e the elementary charge, K_J the Josephson constant and assumed equal to $2e/h$, Φ_0 the magnetic flux quantum, γ_p the proton gyromagnetic ratio, F the Faraday constant, μ_B and μ_N are the Bohr and nuclear magnetons, respectively, V_{90}/V, A_{90}/A and W_{90}/W are the numerical values of the conventional volt, ampere and watt when expressed in terms of the SI volt, ampere and watt, respectively, u is the unified atomic mass unit (also called the dalton, Da), $m_u = m(^{12}\text{C})/12$ is the atomic mass constant and c_1 and c_{1L} are the first radiation constant and first radiation constant for spectral density, respectively.

Technology (CODATA) least-squares adjustment of the values of the constants, the most recent such study available [5]. Because the input data in the 2002 adjustment that determined h or N_A were not as consistent as one would have liked, including results from watt balance and XRCD experiments, it was necessary to weight the *a priori* assigned uncertainty of each such datum by the multiplicative factor 2.325 to obtain an acceptable level of agreement. Although we assume that this difficulty will eventually be sorted out, it has little impact on what is proposed here.

Table 1 gives the relative standard uncertainties u_r of the values of a representative group of constants (including three important conventional electrical units and several energy equivalents) that depend on the unit of mass. The second column gives the uncertainties for these constants resulting from the 2002 CODATA adjustment, which assumes that $m(\mathcal{K}) = 1$ kg exactly; the third and fourth columns give the uncertainties resulting from the same adjustment but with a definition of the kilogram that fixes either h or N_A, respectively.

The first line, which gives u_r of $m(\mathcal{K})$, is included to show explicitly the uncertainty of the mass of the international prototype; this uncertainty, together with the value of $m(\mathcal{K})$ when $m(\mathcal{K})$ is expressed in terms of either of the new mass units, is discussed further below. While many constants not listed in table 1 would have significant reductions in their uncertainties as a result of either new kilogram definition, there are some for which the change would not be zero but would be negligibly small. For example, although current experiments to determine the Newtonian constant of gravitation G require test and field masses calibrated in terms of $m(\mathcal{K})$, because of the large uncertainty involved in such experiments, the 2002 CODATA recommended value of G expressed in terms of either of the newly defined kilograms has an uncertainty only negligibly larger than that of the 2002 value—see section B.2 of appendix B. Similarly, although the Boltzmann constant k and the Stefan–Boltzmann constant σ depend on $m(\mathcal{K})$, they are not included in table 1, because their uncertainties, although smaller in principle, are so dominated by the uncertainty of the molar gas constant R that they remain essentially unchanged by either of the new definitions.

The values of the constants themselves are not given in the table, because when the numerical value chosen for either h or N_A for use in the new definition is exactly equal to its 2002 CODATA value (there is no reason to choose otherwise), the numerical values of *all* of the constants, including those listed in table 1, are equal to their 2002 values, for either of the two new definitions. It is their *uncertainties* that differ and which are of primary interest here, although in either the h-fixed case or N_A-fixed case the values of the constants would be written with additional digits to reflect their now much smaller uncertainties. This is demonstrated in table 2 for a few selected constants. Of course, the uncertainties of those constants that do not depend on $m(\mathcal{K})$ are not changed at all.

Table 1 clearly shows that the uncertainties of the SI values of many constants would be greatly reduced for either new definition, with the reduction depending on the constant and the particular definition adopted. Some uncertainties have been reduced to 0, and others by factors ranging from about 7 to over 1300. For example, the uncertainties of the important practical or 'conventional' electrical units of voltage and current [5] V_{90} and A_{90} (used worldwide for making electrical measurements) expressed in terms of the SI volt V and ampere A, that is, the uncertainties of the ratios V_{90}/V and A_{90}/A, are reduced in the h-fixed case by a factor of 50. In the case where the definition fixes N_A, the u_r of the mass of any particle expressed in the redefined kilogram is identical to the u_r of that particle's mass expressed in the unified atomic mass unit u (also called the dalton, Da)—see section A.2 of appendix A. Moreover, many of these reductions in uncertainty will become even larger in the future when the expected value of the fine-structure constant α from the electron magnetic moment anomaly a_e becomes available with $u_r < 10^{-9}$ [10]. In particular, such a value of α would reduce the numbers 0.17, 0.50, 0.67, 0.83 and 1.2 in table 1 by about a factor of three, since these values of u_r are essentially 1/2, 3/2, 2, 5/2 and 7/2 times $u_r(\alpha)$, respectively. Equally as important, there would be significant reductions in the magnitude of the changes in the recommended values of a large number of constants from one CODATA adjustment to the next. Other benefits of the new definitions are indicated in appendix A, including their effect on the uncertainties of other constants and energy equivalence relations, if some time in the future the ampere were to be redefined so as to fix the value of the elementary charge e and the kelvin were to be redefined so as to fix the value of the Boltzmann constant k.

Table 2. Values of some fundamental constants for the three cases of table 1.

Quantity	Symbol	Numerical value	Unit
		$m(\mathcal{K})$-*fixed (CODATA 2002)*[a]	
Planck constant	h	$6.626\,0693(11) \times 10^{-34}$	J s
Avogadro constant	N_A	$6.022\,1415(10) \times 10^{23}$	mol^{-1}
Electron mass	m_e	$9.109\,3826(16) \times 10^{-31}$	kg
Elementary charge	e	$1.602\,17653(14) \times 10^{-19}$	C
Josephson constant $2e/h$	K_J	$483\,597.879(41) \times 10^{9}$	Hz V^{-1}
		h *fixed*[a]	
Planck constant	h	$6.626\,069\,311 \times 10^{-34}$ (exact)	J s
Avogadro constant	N_A	$6.022\,141\,527(40) \times 10^{23}$	mol^{-1}
Electron mass	m_e	$9.109\,382\,551(61) \times 10^{-31}$	kg
Elementary charge	e	$1.602\,176\,5329(27) \times 10^{-19}$	C
Josephson constant $2e/h$	K_J	$483\,597.879\,13(80) \times 10^{9}$	Hz V^{-1}
		N_A *fixed*[a]	
Planck constant	h	$6.626\,069\,311(44) \times 10^{-34}$	J s
Avogadro constant	N_A	$6.022\,141\,527 \times 10^{23}$ (exact)	mol^{-1}
Electron mass	m_e	$9.109\,382\,5510(40) \times 10^{-31}$	kg
Elementary charge	e	$1.602\,176\,5328(80) \times 10^{-19}$	C
Josephson constant $2e/h$	K_J	$483\,597.879\,14(81) \times 10^{9}$	Hz V^{-1}

[a] The units for the $m(\mathcal{K})$-fixed case (CODATA 2002) are SI units. Although the same unit symbols are used for the other two cases, it should be understood that for the h-fixed case they are units based on fixing the numerical value of h to be equal to that of the 2002 value, while for the N_A-fixed case they are units based on fixing the numerical value of N_A to be equal to that of the 2002 value. (For an explanation of why there is a difference between the last digit of the value of e in the h-fixed and N_A-fixed cases, and similarly for K_J, see section B.3 of appendix B.)

4. Impact of new definitions on the value of $m(\mathcal{K})$

As indicated in appendix B, each of the new definitions introduces a new variable or ‘adjusted constant’ into its respective least-squares adjustment. Essentially, these are just the mass of the prototype expressed in the new mass unit, but it is convenient to write them in terms of the dimensionless ratios $m(\mathcal{K})/(\mathrm{kg})_P$ and $m(\mathcal{K})/(\mathrm{kg})_A$, where $(\mathrm{kg})_P$ and $(\mathrm{kg})_A$ are the units of mass defined by the two alternative definitions, and ‘P’ and ‘A’ are mnemonics for the ‘Planck constant’ and ‘Avogadro constant’, respectively. Each ratio is in fact the numerical value of $m(\mathcal{K})$ when the latter is expressed in terms of the new mass unit.

It can be shown that if the numerical value chosen for either h or N_A to redefine the kilogram is exactly equal to its 2002 CODATA value, then the value of each ratio will be exactly equal to 1, and its u_r will be equal to that of the corresponding 2002 CODATA value of h or N_A. It is the *uncertainties* of these ‘values of 1’ that are of interest here. The values of the two ratios are thus

$$m(\mathcal{K})/(\mathrm{kg})_P = 1.000\,000\,00(17)\ \ [1.7 \times 10^{-7}] \qquad (1)$$

and

$$m(\mathcal{K})/(\mathrm{kg})_A = 1.000\,000\,00(17)\ \ [1.7 \times 10^{-7}], \qquad (2)$$

where the number in parentheses is the standard uncertainty of the last two digits of the quoted value, and the number in square brackets is the relative standard uncertainty u_r. The reason that these uncertainties are the same is because in the 2002 adjustment, the best value of N_A is obtained from the Planck constant h (an adjusted constant) by means of an expression that involves quantities with u_r that are much smaller than $u_r(h)$—see (B6) of appendix B. This is in contrast to the uncertainties of the fundamental constants that depend on $m(\mathcal{K})$—for the same constants, values of u_r that result from the two alternative definitions can differ significantly, as can be seen from table 1.

5. Practical mass measurement system and adoption of a conventional value for $m(\mathcal{K})$

It is evident that the reduced uncertainty of the values of the fundamental constants listed in table 1 would only be achieved at the cost of shifting the current 1.7×10^{-7} relative standard uncertainty of h or N_A to the mass of the international prototype $m(\mathcal{K})$. Thus, if the matter were to be left there the whole enterprise would not be acceptable to the world’s mass-metrology community. The solution we propose is to adopt a conventional value for $m(\mathcal{K})$, designated[1] $m(\mathcal{K})_{07}$, that would be fixed and would serve as the reference standard for the current worldwide ensemble of one-kilogram mass standards, which for Pt–Ir standards has an internal consistency of a few times 10^{-9} kg. More specifically, in terms of the above notation, the conventional value to be adopted would be $m(\mathcal{K})_{07} = 1\,(\mathrm{kg})_P$ exactly or $m(\mathcal{K})_{07} = 1\,(\mathrm{kg})_A$ exactly, depending on the definition of the kilogram selected. This would be analogous to the conventional values of the Josephson and von Klitzing constants, $K_{J\text{-}90} = 483\,597.9\ \mathrm{GHz\,V^{-1}}$ exactly and $R_{K\text{-}90} = 25\,812.807\ \Omega$ exactly, adopted by the International Committee for Weights and Measures (CIPM) to establish practical reference standards for the electrical units [1]. If our suggestion for a redefinition of the kilogram were to be accepted, the mass-metrology community would be in the same position as the electrical metrology community.

In particular, only in those cases where the result of a mass measurement (from an experiment to link mass or force, for example, to fundamental constants) has to be expressed in the SI unit of mass would the result have to be corrected for the experimentally determined difference $m(\mathcal{K})_{07} - m(\mathcal{K})$. Although, initially, this difference would be exactly zero, the 1.7×10^{-7} relative standard uncertainty of the difference would have to be taken into account. To enable this to be done in a coherent way going forward in time, the CIPM could occasionally publish a revised best estimate of the value of $m(\mathcal{K})$, expressed in terms of the new SI mass unit, including the uncertainty of the estimate, based on all of the available data; this estimate and its uncertainty could then be used to determine a correction factor if necessary.

[1] This is for the case when the new definition is adopted in 2007, where the subscript 07 indicates the year; see section 7.

(Again, this would be analogous to what is done in the case of electrical measurements. In those cases where the results of such measurements must be expressed in SI units, corrections are applied based on the current best estimate of the differences between the conventional values of the Josephson and von Klitzing constants and the best estimates of their SI values.) When, in the course of time, the uncertainties of experiments such as the watt balance or XRCD method reach a sufficiently low level so that the SI unit of mass can be realized in practice without reference to a conventional mass standard traceable to $m(\mathcal{K})_{07}$, the international prototype can become a cherished relic of the past.

We have already noted that, in principle, measurements of the SI base quantities amount of substance, electric current and luminous intensity would also be affected by our proposed changes. Since the mole is defined in terms of the number of atoms in 0.012 kg of carbon 12 [1], practical measurements of amount of substance would be in terms of $m(\mathcal{K})_{07}$. However, this would have no significant effect on such measurements due to their comparatively large uncertainties arising from other sources: measurements of amount of substance are generally subject to relative uncertainties many orders of magnitude larger than those considered here. The adoption of either of the new definitions would also have no impact, either now or in the future, on the mass measurement system widely used in physics and chemistry in which the unit of mass is the unified atomic mass unit $\mathrm{u} = m_{\mathrm{u}} = m(^{12}\mathrm{C})/12$ (also called the dalton, Da). The mass $m(^{12}\mathrm{C})$ of the carbon-12 atom in this system would remain $m(^{12}\mathrm{C}) = 12\,\mathrm{u}$ exactly, its molar mass would remain $M(^{12}\mathrm{C}) = 0.012\,\mathrm{kg\,mol^{-1}}$ exactly, and its relative atomic mass $A_{\mathrm{r}}(^{12}\mathrm{C}) = m(^{12}\mathrm{C})/m_{\mathrm{u}} = M(^{12}\mathrm{C})/M_{\mathrm{u}}$ would remain 12 exactly, where m_{u} is the atomic mass constant and M_{u} is the molar mass constant equal to $10^{-3}\,\mathrm{kg\,mol^{-1}}$ exactly. This system is used to determine with very small uncertainties, that is, with values of u_{r} as small as a few times 10^{-10} or even less, the mass of atomic-size or 'microscopic' bodies such as fundamental particles, atoms and molecules. With regard to electric current, as discussed in connection with table 1, the effect of the proposed changes would be beneficial, because measurements of electric current are already linked to fundamental constants through the Josephson and quantum Hall effects and K_{J} and R_{K}. And finally, as regards luminous intensity, the uncertainties of measurements of this and related quantities are sufficiently large that the effects of possible differences between $m(\mathcal{K})_{07}$ and $m(\mathcal{K})$ would be totally insignificant.

Based on all of the discussion of this section, we believe that even if it were to be eventually discovered that the value of h or N_{A} chosen to redefine the kilogram were such that $(m(\mathcal{K})_{07} - m(\mathcal{K}))/m(\mathcal{K}) \approx 10^{-6}$, which the current difference between the watt balance and XRCD results might lead one to believe is a possibility, the consequences could be better dealt with through a redefinition now. For if we do not revise the definition of the kilogram it may be necessary to make a substantial revision to the values of both h and N_{A}, with consequent changes to many of the other fundamental constants. But by changing the definition of the kilogram now the values of h and N_{A} may be kept unchanged regardless of any new experimental results. Instead the CIPM could simply publish a revised value of the conventional mass of the prototype, which might for example be named $m(\mathcal{K})_{11}$ if this occurred in 2011.

6. The need to continue current experiments

We should like to emphasize that redefining the kilogram as proposed here would in no way diminish the importance of any of the several experiments underway in various laboratories around the world to determine h and N_{A} with $u_{\mathrm{r}} \approx 10^{-8}$. On the contrary, the fact of having redefined the kilogram in terms of a fundamental constant would require appropriate practical means to measure the mass of the international prototype $m(\mathcal{K})$ in terms of the new definition. Thus, although one of the goals of such experiments would change, that of determining the value of a fundamental constant with unprecedented accuracy, it would be replaced by that of determining the mass $m(\mathcal{K})$. (Because of (B6) in appendix B, the change in goals would to a great extent apply even for determinations of the value of the constant not chosen to define the kilogram.) Of course, the other main purpose of such experiments—to eventually develop a method that would enable the SI unit of mass to be realized by anyone at anytime and at anyplace with the required uncertainty—would remain unchanged. Researchers carrying out these experiments would, therefore, still have every reason to pursue their work as vigorously as possible.

7. Conclusion

The implementation of a definition of the kilogram that fixes either the value of h or N_{A} would immediately reduce the uncertainties of the SI values of many fundamental constants by significant factors, with further reductions as experiment and theory advance. The vast majority of the world's measurements of mass would be unaffected by such a redefinition, because by adopting a conventional value for the mass of the international prototype, $m(\mathcal{K})_{07} = 1\,\mathrm{kg}$ exactly, it could remain the basis for the worldwide system of practical mass measurement. Only in unusual circumstances would it be necessary to take into account the difference between $m(\mathcal{K})_{07}$ and the newly defined SI unit of mass. We strongly believe that there is no reason to postpone this decision, for example, to wait until the mass of the international prototype can be related to either of these constants with a relative standard uncertainty $u_{\mathrm{r}} \approx 10^{-8}$. We see many advantages in putting a new definition into place now, when doing so will immediately reduce the uncertainties of the SI values of many fundamental constants as well as the SI values of the widely used conventional electrical units discussed above. From a purely scientific point of view, it is quite possible that the lifting of the veil of unnecessarily large uncertainty from the values of many quantum-physics-related constants will stimulate new experimental and theoretical work directed at testing the fundamental theories of physics.

Because there are different advantages to choosing a definition that fixes h or one that fixes N_{A}, we leave the choice between these alternatives for further discussion by the appropriate international committees. We suggest possible wordings for either definition in appendix A, where we also review their relative merits. Our hope is that the 23rd CGPM, which convenes in October of 2007, will adopt one of the new definitions, basing the numerical value to be used in the new definition on the best data available at the time. This value could, in fact, be the 2006 CODATA value, which should be available by the time of the 23rd CGPM.

Appendix A. Possible words for new kilogram definitions, and their respective advantages

Ways of redefining the kilogram along the lines discussed in this paper have already appeared in a number of publications [4, 11–22], including suggested words for a revised definition. Some possible phrasings for a new definition are presented below, first for a definition that fixes h and then for a definition that fixes N_A. We also review the relative merits of the two alternatives, but since the roles the new definitions play in reducing the uncertainties of the fundamental constants are summarized in table 1 and its associated text, we do not repeat these arguments below. Also, throughout this appendix, the defining numerical values are based on the 2002 CODATA set of recommended values of the constants (but see section B.3 of appendix B for an explanation of the number of digits used) [5]; and it should be recalled that the current SI definition of the metre has the effect of fixing the speed of light in vacuum c to be exactly 299 792 458 m s^{-1} [1].

Appendix A.1. Definitions that fix the value of the Planck constant h

We suggest three alternative wordings, labelled as (h-1), (h-2) and (h-3). These are all in effect the same definition, although presented in rather different ways. In this regard, it is important to recognize that any definition that fixes the value of the Planck constant h, which of course is an invariant of nature, establishes an invariant unit of mass. This is because (following the notation introduced in section 4 and that used in appendix B) $h = \{h\}_P\,J_P\,s = \{h\}_P\,m^2\,(kg)_P\,s^{-1}$, where here $\{h\}_P$ is the numerical value of the adopted value of h. (The subscript P on the joule unit symbol indicates that it is the joule in the new unit system.) Thus $(kg)_P = (h/\{h\}_P)\,m^{-2}\,s$, and since both h and $\{h\}_P$ are invariants, and the unit metre, m, and the unit second, s, are themselves defined in terms of invariants, $(kg)_P$ must also be an invariant.

(h-1) The kilogram is the mass of a body at rest such that the value of the Planck constant h is exactly $6.626\,069\,311 \times 10^{-34}$ joule second.

(h-2) The kilogram is the mass of a body at rest whose equivalent energy corresponds to a frequency of exactly $[(299\,792\,458)^2/6\,626\,069\,311] \times 10^{43}$ hertz.[2]

(h-3) The kilogram is the mass of a body whose de Broglie wavelength is exactly $6.626\,069\,311 \times 10^{-34}$ m when moving with a velocity of exactly one metre per second.

Definition (h-2) fixes h through the combination of the Einstein relation $E = mc^2$ and the relation $E = h\nu$ first applied by Planck to the emission and absorption of radiation and subsequently by Einstein to the energy of photons [14], while definition (h-3) fixes h through the de Broglie relation $\lambda = h/p = h/mv$.

[2] An equivalent definition that is simpler numerically might read as follows: The kilogram is the mass of a body at rest whose equivalent energy is equal to that of $299\,792\,458 \times 10^{27}$ optical photons of wavelength in vacuum of 662.606 931 1 nanometres.

The reasons for preferring a definition of the kilogram that fixes h include the following.

1. The Planck constant is the fundamental constant of quantum mechanics just as the speed of light is the fundamental constant of relativity. A definition of the kilogram that fixes the value of h is, therefore, a complement to the current definition of the metre which fixes the value of c, and a definition that fixes h means that the constants appearing in the fundamental relations $E = mc^2$, $E = h\nu$ and $\lambda = h/p$ all have exactly known values.
2. The uncertainties of eight CODATA recommended energy equivalence relations that involve only h, or h and c, are completely eliminated.
3. Under the assumption that the watt balance eventually achieves its uncertainty goal of $u_r \approx 10^{-8}$, it could be used to directly calibrate unknown standards of mass without the uncertainty of any other constant contributing to the uncertainty of the calibration.
4. If the ampere were to be redefined so as to fix the value of the elementary charge e, for example, by defining it as the flow of a specified number of electrons per second (thereby making the magnetic constant μ_0 and the electric constant ε_0 quantities to be determined by experiment), then a number of other constants would become exactly known, including the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$. The uncertainties of the four CODATA recommended energy equivalence relations that involve only e and h, or e, h and c, would also vanish.
5. If, as a result of the above redefinition, the value of e were exactly known, then since the value of the Josephson constant $K_J = 2e/h$ and the von Klitzing constant $R_K = h/e^2$ would also be exactly known, the need for the conventional values $K_{J\text{-}90}$ and $R_{K\text{-}90}$, their implied conventional units V_{90}, Ω_{90} and A_{90}, and other related conventional electrical units would be eliminated. This would simplify the realization of SI electrical units and the relationship between electrical measurements and the fundamental constants. (It is interesting to note that $K_{J\text{-}90}$ and $R_{K\text{-}90}$ can be viewed as defining fixed, conventional values e_{90} and h_{90} of the elementary charge and Planck constant, respectively, via the relations $e_{90} = 2/(K_{J\text{-}90}R_{K\text{-}90})$ and $h_{90} = 4/(K_{J\text{-}90}^2 R_{K\text{-}90})$. When one measures a current in terms of the Josephson and quantum Hall effects using the conventional values $K_{J\text{-}90}$ and $R_{K\text{-}90}$, one is actually measuring it in terms of e_{90} per second.)
6. Similarly, if the kelvin were to be redefined so as to fix the value of the Boltzmann constant $k = R/N_A$, where R is the molar gas constant, then the uncertainty of the Stefan–Boltzmann constant $\sigma = (2\pi^5/15)k^4/(h^3c^2)$ would be zero, as would the uncertainties of the four CODATA recommended energy equivalence relations involving only k and h, or k, h and c.

Appendix A.2. Definitions that fix the value of the Avogadro constant N_A

A definition of the kilogram that fixes N_A has interesting consequences because of the relationship between the mass of the carbon-12 atom $m(^{12}\text{C})$ and the Avogadro constant N_A through the definition of the mole. The latter reads in part 'The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12' [1]. The Avogadro constant (SI unit mol^{-1}) is defined according to $N_\text{A} = M(X)/m(X)$, where $M(X)$ is the molar mass of entity X (i.e. the mass per amount of substance of X, SI unit kg mol^{-1}) and $m(X)$ is the mass of X (SI unit kg). Thus, (i) the number of entities in one mole of X is (N_A mol), (ii) $(N_\text{A}\,\text{mol})m(^{12}\text{C}) = 0.012\,\text{kg}$ exactly and (iii) $M(^{12}\text{C})$, the molar mass of carbon 12, is exactly $0.012\,\text{kg mol}^{-1}$.

Again, we suggest three alternative wordings, labelled (N_A-1), (N_A-2) and (N_A-3), each of which in its own way fixes the value of the Avogadro constant. To see how fixing the value of N_A (definition (N_A-1) does this explicitly) establishes an invariant unit of mass, we note that the relation given in (ii) of the above paragraph (again following the notation of section 4 and appendix B) may be written as $(N_\text{A}\,\text{mol}_\text{A})m(^{12}\text{C}) = 0.012\,(\text{kg})_\text{A}$. (The subscript A on the mole unit symbol indicates that it is the mole in the new unit system.) Since $(N_\text{A}\,\text{mol}_\text{A})$ is an adopted exact number, and the mass $m(^{12}\text{C})$ is an invariant of nature and 0.012 is a fixed exact number, $(\text{kg})_\text{A}$ is also an invariant.

(N_A-1) The kilogram is the mass of a body at rest such that the value of the Avogadro constant N_A is exactly $6.022\,141\,527 \times 10^{23}$ inverse mole.

(N_A-2) The kilogram is the mass of exactly $5.018\,451\,272\,5 \times 10^{25}$ unbound carbon-12 atoms at rest and in their ground state.

(N_A-3) The kilogram is the mass of exactly $(6.022\,141\,527 \times 10^{23}/0.012)$ unbound carbon-12 atoms at rest and in their ground state.

To see how definition (N_A-2) fixes the value of N_A, we note that it implies $m(^{12}\text{C}) = 1\,\text{kg}/(5.018\,451\,272\,5 \times 10^{25})$, which together with the expression $(N_\text{A}\,\text{mol})m(^{12}\text{C}) = 0.012\,\text{kg}$ leads to $N_\text{A} = (0.012 \times 5.018\,451\,272\,5 \times 10^{25})\,\text{mol}^{-1} = 6.022\,141\,527 \times 10^{23}\,\text{mol}^{-1}$. Definition ($N_\text{A}$-3) fixes N_A in the same way. Thus, both definitions (N_A-2) and (N_A-3) lead to a simplified definition of the mole, which might read as follows.

> The mole is the amount of substance of a system that contains exactly $6.022\,141\,527 \times 10^{23}$ specified entities.

It should be noted that none of the proposed definitions that fix the value of N_A alter the exact values $m(^{12}\text{C}) = 12\,\text{u}$, $M(^{12}\text{C}) = 0.012\,\text{kg mol}^{-1}$ and $A_\text{r}(^{12}\text{C}) = 12$, where $A_\text{r}(X)$ is the relative atomic mass of X (see section 5).

The following are among the reasons for preferring a definition of the kilogram that fixes N_A, in particular, either (N_A-2) or (N_A-3).

1. It is simple, conceptually, enabling it to be widely understood.
2. It allows the mole to be redefined in a simpler and more understandable way.
3. It fixes the value of the unified atomic mass unit u (also called the dalton, Da), since $\text{u} = m_\text{u} = m(^{12}\text{C})/12 = M_\text{u}/N_\text{A}$, where m_u is the atomic mass constant and M_u is the molar mass constant and is equal to $10^{-3}\,\text{kg mol}^{-1}$ exactly.
4. Because of point 3, the relative uncertainty of the mass of a body expressed in the new mass unit is the same as that of the mass of the body expressed in u. Also, because of point 3, the uncertainties of the four CODATA recommended energy equivalence relations that involve only m_u, or m_u and c, completely vanish.
5. If, as above, the ampere were to be redefined so as to fix the value of the elementary charge e, the value of the Faraday constant F would be exactly known since $F = N_\text{A}e$. The uncertainties of the two CODATA recommended energy equivalence relations that involve only e, m_u and c would also become exactly known.
6. If, as above, the kelvin were to be redefined so as to fix the value of the Boltzmann constant k, then the molar gas constant $R = kN_\text{A}$ would have an exact value, as would the two CODATA recommended energy equivalence relations that involve only k, m_u and c.

Appendix B. Determining best values of the fundamental constants based on a definition of the kilogram that fixes the value of h or N_A

Best values of the fundamental constants in SI units, as obtained from the Committee on Data for Science and Technology (CODATA) 2002 least-squares adjustment of the values of the constants, have recently been recommended by CODATA [5]. The 2002 adjustment, carried out by two of the authors (PJM, BNT) under the auspices of the CODATA Task Group on Fundamental Constants, took into account all relevant data available by 31 December 2002, plus selected data that appeared by the Fall of 2003. The objective of this appendix is to describe the modifications to the 2002 least-squares adjustment that need to be made to account for a new definition of the kilogram, and to obtain the values of the fundamental constants that result from such a modified adjustment.

Appendix B.1. Least-squares adjustments and observational equations: general

The numerical values of the fundamental constants depend on the units in which the values of the constants are expressed. In order to find the effect that changing the definition of the kilogram would have on these numerical values, it is useful to review first the effect that such a change would have on the SI units. This can be done for an arbitrary change in the kilogram without initially specifying the form of its redefinition.

If the international prototype $\mathcal{K}$ were to be replaced by a new object $\mathcal{K}'$ and the kilogram were to be redefined to be the mass of $\mathcal{K}'$, with $m(\mathcal{K}) = \Lambda m(\mathcal{K}')$, where Λ is a dimensionless numerical factor (a relation that may also be written as $1\,\text{kg} = \Lambda\,\text{kg}'$), then there would be changes in other SI units, including SI base units, as a consequence of the definitions of these units. These changes are summarized in table B1, where the primed units represent the SI units in the new system of units in which the unit of mass is kg′, and the relations between units that depend on the unit of mass contain the factor Λ raised to a power.

In order to take into account the interaction between units and values of the fundamental constants, we use the

Table B1. Changes in SI base units and in relevant SI derived units corresponding to a change in the definition of the kilogram.

SI base unit changes	SI derived unit changes
$1\,\mathrm{m} = 1\,\mathrm{m}'$	$1\,\mathrm{Hz} = 1\,\mathrm{Hz}'$
$1\,\mathrm{kg} = \Lambda\,\mathrm{kg}'$	$1\,\mathrm{N} = \Lambda \mathrm{N}'$
$1\,\mathrm{s} = 1\,\mathrm{s}'$	$1\,\mathrm{J} = \Lambda\,\mathrm{J}'$
$1\,\mathrm{A} = \Lambda^{1/2}\,\mathrm{A}'$	$1\,\mathrm{C} = \Lambda^{1/2}\,\mathrm{C}'$
$1\,\mathrm{K} = 1\,\mathrm{K}'$	$1\,\mathrm{V} = \Lambda^{1/2}\,\mathrm{V}'$
$1\,\mathrm{mol} = \Lambda\,\mathrm{mol}'$	$1\,\Omega = 1\,\Omega'$
$1\,\mathrm{cd} = \Lambda\,\mathrm{cd}'$	$1\,\mathrm{T} = \Lambda^{1/2}\,\mathrm{T}'$

conventional notation in which a physical quantity A is written as $A = \{A\}\,[A]$, where $\{A\}$ is the numerical value of A when A is expressed in the unit $[A]$. Here, it is assumed, however, that $[A]$ is the coherent SI unit for A. The modified SI unit that would result from a change in the definition of an SI base unit is denoted $[A]'$, with a corresponding changed numerical value $\{A\}'$ that satisfies the relation $\{A\}\,[A] = \{A\}'\,[A]'$. (In our case the redefined unit is the kilogram, but the treatment in most of the following two paragraphs may be viewed as more general than this and the prime as applying to any redefined base unit.)

In a least-squares adjustment of the constants, such as the 2002 CODATA adjustment, the observational equations, that is, the theoretical relations between the measured and calculated input data and the variables or 'adjusted constants', are of the form (see appendix E of [23])

$$q_i \doteq f_i(z_1, z_2, \ldots, z_M), \qquad i = 1, 2, \ldots, N, \qquad \text{(B1)}$$

where q_i is the ith datum of the N input data and z_j is the jth of M constants. Here, the constants z_j may include fixed constants, adjusted constants or constants that are functions of adjusted constants. The symbol $\doteq$ denotes the fact that the two sides of the equation are equal in principle but not numerically, because the set of equations is overdetermined. However, in a least-squares adjustment, the calculations are done with only the numerical values of quantities, so (B1) can be interpreted as representing the equivalent numerical-value equation

$$\{q_i\} \doteq f_i(\{z_1\}, \{z_2\}, \ldots, \{z_M\}) \qquad i = 1, 2, \ldots, N, \qquad \text{(B2)}$$

which is the actual equation used in the least-squares adjustment computer code. Equations (B1) and (B2) are equivalent due to the fact that all quantities are expressed in coherent SI units so that the units $[q_i]$ and $[f_i(z_1, z_2, \ldots, z_M)]$ are the same, together with the fact that

$$f_i(\{z_1\}, \{z_2\}, \ldots, \{z_M\}) = \{f_i(z_1, z_2, \ldots, z_M)\}, \quad i = 1, 2, \ldots N, \qquad \text{(B3)}$$

which follows from the coherence of the SI base and derived units.

For the present analysis, a modification of the observational equations is necessary. In particular, the measured and calculated input data are known in coherent SI units, while the objective is to determine the values of the constants in the new coherent units, that is, the 'primed' units. This yields for each observational equation a conversion factor $[q_i]'/[q_i]$ that depends on the dimension of q_i, and leads to new observational equations of the form

$$\{q_i\} = \frac{[q_i]'}{[q_i]} f_i(\{z_1\}', \{z_2\}', \ldots, \{z_M\}'), \qquad i = 1, 2, \ldots, N, \qquad \text{(B4)}$$

where it should be recognized from the above discussion that $[q_i]' = [f_i(z_1, z_2, \ldots, z_M)]'$. Since all of the transformations between the old and new units involve Λ raised to a power (although in some cases the power is zero—see table B1), the coefficient $[q_i]'/[q_i]$ of f_i in (B4) will also be Λ raised to some power.

Another modification of the least-squares analysis is required, because either of the new definitions of the kilogram fixes the value of a fundamental constant, with the result that there is effectively one less adjusted constant. However, the reduction is offset by making $\Lambda = m(\mathcal{K})/\mathrm{kg}'$ an adjusted constant; the value of this variable is needed to determine the value of $m(\mathcal{K})$ in the new unit system. The new definition of the kilogram that fixes the value of the Planck constant h is implemented in the adjustment by assigning an exact value to $\{h\}'$, namely the 2002 CODATA value

$$\{h\}' = \{h\}_\mathrm{P} = 6.626\,069\,311 \times 10^{-34}, \qquad \text{(B5)}$$

where we have replaced the prime by the mnemonic P for Planck constant to make it clear that we are dealing with the redefinition of the kilogram that fixes h. (For this same reason, in the main text, in appendix A, and in the remainder of this appendix, when appropriate, kg′ is replaced by $(\mathrm{kg})_\mathrm{P}$, or in the fixed-N_A case by $(\mathrm{kg})_\mathrm{A}$, where A is a mnemonic for Avogadro constant.)

The new definition that fixes the value of the Avogadro constant N_A rather than the Planck constant h is implemented indirectly, because unlike h, N_A is not an adjusted constant in the 2002 least-squares adjustment; its 2002 recommended value was calculated from the values of the adjusted constants that resulted from the final 2002 least-squares adjustment using the relation

$$N_\mathrm{A} = \frac{c}{2}\frac{A_\mathrm{r}(\mathrm{e})\alpha^2}{R_\infty}\frac{M_\mathrm{u}}{h}, \qquad \text{(B6)}$$

where $c = 299\,792\,458\,\mathrm{m\,s^{-1}}$ exactly is the speed of light in vacuum, $M_\mathrm{u} = 10^{-3}\,\mathrm{kg\,mol^{-1}}$ exactly is the molar mass constant, and the adjusted constants $A_\mathrm{r}(\mathrm{e})$, α and R_∞ are the relative atomic mass of the electron, the fine-structure constant and the Rydberg constant, respectively. (Note that none of the quantities in this equation except h and N_A depend on $m(\mathcal{K})$.) In the 2002 least-squares adjustment, any of the four adjusted constants in (B6) could have been replaced by another with the aid of that equation. To obtain the observational equations for the fixed-N_A case, one uses (B6) and takes $\{N_\mathrm{A}\}' = \{N_\mathrm{A}\}_\mathrm{A} = 6.022\,141\,527 \times 10^{23}$, which is the 2002 value of N_A.

Appendix B.2. Least-squares adjustments and observational equations: details

The previous section discusses in a rather general way the modifications that must be made to the 2002 least-squares adjustment to account for a new definition of the kilogram that fixes either the Planck constant h or Avogadro constant

N_A. Here, those specific observational equations that are modified by a change in the definition of the kilogram are written explicitly.

We first consider the Newtonian constant of gravitation G. Because G has no known relationship with any other constant, and the measurements of G considered in the 2002 adjustment had no significant correlations with any of the other input data, for simplicity the 2002 recommended value of G was obtained from a separate least-squares adjustment of the individual measurements of G, for which the observational equations were simply $G \doteq G$. For equations of this form, a least-squares adjustment amounts to a weighted mean of the individual values. However, as implied in section 3, all the measurements of G on which the 2002 CODATA recommended value is based employed test and field masses calibrated in terms of $m(\mathcal{K})$. Thus, if the kilogram were to be redefined by fixing either h or N_A, based on the discussion in section B.1, the appropriate observational equations for these individual values would be

$$\{G\} \doteq \Lambda\{G\}'. \qquad \text{(B7)}$$

Nevertheless, the structure of the least-squares adjustment would remain such that the individual values of G would have no impact on the value of any quantity other than G, with the result that $\Lambda\{G\}' = \{G_{02}\}$, where G_{02} is the 2002 CODATA value of G. Since, as determined from the rest of the adjustment, Λ is equal to 1 (a consequence of choosing $\{h\}'$ and $\{N_A\}'$ to have their 2002 values) and $u_r(\Lambda)$ is nearly three orders of magnitude smaller than $u_r(G_{02})$, this last equation implies that for all practical purposes, the 2002 value of G and its uncertainty are unchanged by either new definition, as pointed out in section 3.

We next consider the observational equations for the data related to the Rydberg constant as given in table XIX of [5]. In fact, none of these equations is affected by a redefinition of the kilogram, because none of them involves the unit of mass.

Finally, we consider the observational equations for the other data as given in table XXI of [5], a number of which do in fact depend on the kilogram. These are the equations for the proton gyromagnetic ratio determined by the high-field method $\Gamma'_{p\text{-}90}(\text{hi})$; the Josephson constant K_J, assumed to be equal to $2e/h$, where e is the elementary charge; the product $K_J^2 R_K = 4/h$, where R_K is the von Klitzing constant, assumed to be equal to h/e^2; the Faraday constant F_{90} and the molar volume of silicon $V_m(\text{Si})$. Their respective observational equations in table XXI of [5] are $B27$, $B29$, $B31$, $B32$ and $B46$. It is important to recognize that all of the present discussion rests to a great extent on the validity of the assumptions that K_J and R_K are linked to the fundamental constants by these relations.

Two of these equations, $B27$ for $\Gamma'_{p\text{-}90}(\text{hi})$ and $B32$ for F_{90}, contain the conventional values of the Josephson and von Klitzing constants, $K_{J\text{-}90} = 483\,597.9 \times 10^9\,\text{Hz}\,\text{V}^{-1}$ exactly and $R_{K\text{-}90} = 25\,812.807\,\Omega$ exactly [1,5] ($K_{J\text{-}90}$ and $R_{K\text{-}90}$ are examples of z_j in (B1) being a fixed constant). These values were adopted by the CIPM, at the request of the CGPM, for worldwide use starting 1 January 1990 to ensure the international compatibility of electrical measurements (see section 5). The SI unit of $K_{J\text{-}90}$, $\text{Hz}\,\text{V}^{-1}$, depends on the kilogram and hence $K_{J\text{-}90}$ requires special consideration. In order to have the same numerical value in the new unit system as that of $K_{J\text{-}90}$, $483\,597.9 \times 10^9$, it is convenient to define a modified conventional Josephson constant given by $K'_{J\text{-}90} = \{K_{J\text{-}90}\}[K_{J\text{-}90}]' = \Lambda^{1/2} K_{J\text{-}90}$ so that $\{K'_{J\text{-}90}\}' = \{K_{J\text{-}90}\}$. We can then replace $K_{J\text{-}90}$ by $\Lambda^{-1/2} K'_{J\text{-}90}$ in the observational equations and use the current conventional numerical value. On the other hand, no special consideration is necessary for $R_{K\text{-}90}$; $R'_{K\text{-}90} = R_{K\text{-}90}$, because its SI unit, Ω, is independent of the kilogram and hence is unchanged if the kilogram is redefined.

Table B2. Observational equations that express the input data used in the 2002 CODATA least-squares constants adjustment that depend on the mass of the international prototype as functions of the adjusted constants (the Newtonian constant of gravitation G excepted) for the case where the kilogram is redefined.

Type of input datum[a]	Observational equation
$B27'$	$\{\Gamma'_{p\text{-}90}(\text{hi})\} \doteq -\left\{\dfrac{\Lambda c[1+a_e(\alpha,\delta_e)]\alpha^2}{K'_{J\text{-}90} R_{K\text{-}90} R_\infty h}\left(\dfrac{\mu_e}{\mu'_p}\right)^{-1}\right\}'$
$B29'$	$\{K_J\} \doteq \left\{\left(\dfrac{8\Lambda\alpha}{\mu_0 c h}\right)^{1/2}\right\}'$
$B31'$	$\{K_J^2 R_K\} \doteq \left\{\dfrac{4\Lambda}{h}\right\}'$
$B32'$	$\{F_{90}\} \doteq \left\{\dfrac{\Lambda c M_u A_r(e)\alpha^2}{K'_{J\text{-}90} R_{K\text{-}90} R_\infty h}\right\}'$
$B46'$	$\{V_m(\text{Si})\} \doteq \left\{\dfrac{\sqrt{2}\Lambda c M_u A_r(e)\alpha^2 d_{220}^3}{R_\infty h}\right\}'$

[a] The numbers in the first column correspond to the numbers in the first column of table XXI of [5]. For simplicity, the function $a_e(\alpha, \delta_e)$ is not explicitly given.

Table B2 gives the modified observational equations for the five types of input data that depend on the kilogram (other than G), to which the following comments apply: (i) the numbering of the equations is as in table XXI of [5], but with a prime to differentiate between these new observational equations and their counterparts in table XXI; (ii) the subscript 90 on $\Gamma'_{p\text{-}90}(\text{hi})$ and F_{90} indicates that their values are determined using the '1990' conventional electrical units; (iii) $a_e(\alpha, \delta_e)$ is the theoretical expression for the electron magnetic moment anomaly and is a function of the fine-structure constant α and δ_e, where the latter adjusted constant accounts for the uncertainty of the expression, μ_e/μ'_p is the electron to shielded proton magnetic moment ratio, μ_0 is the magnetic constant and is equal to $4\pi \times 10^{-7}\,\text{N}\,\text{A}^{-2}$ exactly (it is independent of the kilogram although its value is fixed by the definition of the ampere), and d_{220} is the {220} lattice spacing of a pure, single crystal of naturally occurring silicon at a specified temperature and pressure; and (iv) the corresponding observational equations used in the 2002 adjustment can be easily recovered from those given in table B1 by setting $\Lambda = 1$ and deleting the prime everywhere it appears. In that case, the adjusted constants in these equations are α, δ_e, R_∞, h, μ_e/μ'_p and d_{220}, which may be compared to the adjusted constants in the present case, α, δ_e, R_∞, Λ, μ_e/μ'_p and d_{220}.

Appendix B.3. Calculation of new values of the constants

The required new adjustments, as well as the calculation of best values of other constants from the resulting adjusted constants, were done following the procedures used in the 2002 CODATA adjustment. However, as a practical matter, in order to use the exact same least-squares formalism and computer code that was employed in the 2002 adjustment, the calculations for the fixed-h case were carried out by including the following additional observational equation:

$$6.626\,069\,311\,000\,00(10) \times 10^{-34} = \{h\}_{\mathrm{P}}. \qquad \text{(B8)}$$

This equation assigns to the numerical value of h in the fixed-h case the same numerical value as that of the 2002 value, but with its uncertainty reduced by about six orders of magnitude. As far as the adjustments are concerned, this reduction is sufficiently large to make h an exact quantity. In (B8), we have included two additional digits for the fixed value of h, and also in (B9) below for the fixed value of N_{A}, beyond those given in [5] in order to ensure that the numerical value of each adjusted and derived constant from either of the new adjustments is essentially the same as that of its 2002 recommended value. Rounding to the number of digits provided in [5] leads to significant inequalities, and also introduces inconsistencies in the more accurate values of the constants resulting from either new kilogram definition. In general, for all of the constants given in [5], the added digits in (B8) and (B9) are sufficient to provide results that are consistent with the 2002 recommended values within one in the last digit shown in the conventional two-digit-uncertainty format used to report their values. It is also sufficient to provide such consistency between the values of the same constants resulting from the two different definitions, as is evident in table 2.

The calculations for the fixed N_{A} case were carried out in a similar manner. However, the observational equation used to fix the numerical value of N_{A} to be the same as that of its 2002 value was, following the discussion in section B1 (see (B6)),

$$6.022\,141\,527\,000\,00(10) \times 10^{23} \doteq \left\{ \frac{c}{2} \frac{A_{\mathrm{r}}(\mathrm{e})\alpha^2}{R_\infty} \frac{M_{\mathrm{u}}}{h} \right\}_{\mathrm{A}}. \qquad \text{(B9)}$$

The values of Λ for the two cases are given in (1) and (2) of section 4. Although these equations were written down by inspection, they have been verified by the least-squares adjustments. We have also recalculated all of the tables of recommended values that are given in [5], that is, tables XXV through XXXII, for both the fixed-h and fixed-N_{A} cases, but we believe that for our purposes here, tables 1 and 2 of section 3 should provide the reader with a reasonable sense of the results. Those desiring additional information may contact the authors.

References

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[1] BIPM 1998 *The International System of Units* 7th edn; and *Supplement 2000: Addenda and Corrigenda to the 7th Edition (1998)* (Sèvres, France: Bureau International des Poids et Mesures) available at http://www.bipm.org

[2] Davis R S 2003 *Metrologia* **40** 299–305

[3] Quinn T J 1991 *IEEE Trans. Instrum. Meas.* **40** 81–5

[4] Taylor B N 1991 *IEEE Trans. Instrum. Meas.* **40** 86–91

[5] Mohr P J and Taylor B N 2005 *Rev. Mod. Phys.* **77** 1–106

[6] Kibble B P 1975 *Atomic Masses and Fundamental Constants* vol 5, ed J H Sanders and A H Wapstra (New York: Plenum) pp 545–51

[7] Eichenberger A, Jecklemann B and Richard P 2003 *Metrologia* **40** 356–65

[8] Deslattes R D *et al* 1974 *Phys. Rev. Lett.* **36** 463–6

[9] Becker P 2003 *Metrologia* **40** 366–75

[10] Gabrielse G and Kinoshita T 2004 private communications

[11] Taylor B N 1989 *IEEE Trans. Instrum. Meas.* **38** 164–6

[12] Wignall J W G 1992 *Phys. Rev. Lett.* **68** 5–8

[13] Taylor B N 1994 *Metrologia* **31** 181–94

[14] Taylor B N and Mohr P J 1999 *Metrologia* **36** 63–4

[15] Taylor B N and Mohr P J 2001 *IEEE Trans. Instrum. Meas.* **50** 563–7

[16] Göbel E O 2001 *Phys. Bl.* **57** 35–41

[17] Kose V, Siebert B R L and Wöger W 2003 *Metrologia* **40** 146–53

[18] Davis R S 2004 private communication

[19] Robinson I A 2004 *Phys. World* **17** (5) 31–5

[20] Bordé C J 2004 *C. R. Phys.* **5** 813–20

[21] Schwitz W, Jeckelmann B and Richard P 2004 *C. R. Phys.* **5** 881–92

[22] Flowers J 2004 *Science* **306** 1324–30

[23] Mohr P J and Taylor B N 2000 *Rev. Mod. Phys.* **72** 351–495