The research in the past year focused on two objectives. The first one is pertaining to fundamental understanding of transport phenomena (hydrodynamics and mass transfer) in reverse osmosis membrane based wastewater treatment. The second one is systematic analysis and optimization of pressure retarded osmosis that may be used to generate power from combining saltwater and low-salinity wastewater.

Based on the CFD results obtained in the previous report, a system-level model was developed below to describe the coupled transport phenomena in a wastewater process in West Basin Water District that uses 2-stage RO design:

\[
\frac{dQ}{dx} = -J_w A_m, \quad @x = 0, \quad Q = Q_0
\]

\[
\frac{d(\Delta P)}{dx} = -k_2 Q^{n_f}, \quad @x = 0, \quad \Delta P = \Delta P_0
\]

\[
J_w = L_p [\Delta P - \pi \exp (J_w/(k_3 Q^{n_c}))]
\]

\[
\pi = Q_0 \pi_0 / Q
\]

Following the least-squares regression approach, the model parameters were determined by using MATLAB. An almost perfect match between model and literature data from Hydranautics was obtained by using the parameters below: \( L_p = 0.176 \, \text{gfd/psi}, \; k_2 = 9.718 \times 10^{-4} \, \text{psi/gpm}^{-1.38} \) (first stage), \( k_3 = 0.0103 \, \text{m/s/gpm}^{0.4} \) (first stage), and \( n_f = 1.38 \).

![Fig. 1](image1.png)  
(a)  
(b)  

**Fig. 1.** Comparison between computational model of 2-stage RO wastewater treatment and literature data from Hydranautics.

For single- and multi-stage PRO design configurations (shown in Fig. 2), an optimization model was formulated and solved to maximize power generation from salinity gradient. The optimal results are shown in Fig. 3. It is seen that the specific power generation normalized by the draw solution feed osmotic pressure (NSEP) increases with the number of stages as well as a dimensionless parameter \( \gamma_{tot} = A_{tot} L_p \pi_0 / Q_0 \). As compared to the single-stage PRO, the multi-stage arrangement not only increases flux and volume gain (\( q_{tot} - 1 \)), but also allows a stage-dependent, progressively decreasing hydraulic pressure, both of which contribute to enhanced specific power generation and power density.

![Fig. 2](image2.png)  

**Fig. 2.** Schematic of PRO power generation combing salt water and low-salinity wastewater.
\[
\max_{\alpha_j, \gamma_j, q_j} \text{SEP} = \sum_{j=1}^{N} \frac{q_j - 1}{\alpha_j}
\]
\[
s.t.
0 = \gamma_j - \alpha_j \left(1 - q_j + \alpha_j \ln \frac{\alpha_j - 1}{\alpha_j - q_j}\right), j = 1, \ldots, N
\]
\[
0 = \gamma_{\text{tot}} - \gamma_1 - \sum_{j=2}^{N} \gamma_j \prod_{k=1}^{j-1} q_k
\]
\[
0 \leq \alpha_{j+1}q_j - \alpha_j, j = 1, \ldots, N - 1
\]
\[
0 \leq \gamma_j, j = 1, \ldots, N
\]
\[
0 \leq \alpha_j - 1, j = 1, \ldots, N
\]
\[
0 \leq q_j - 1, j = 1, \ldots, N
\]

Fig. 3. (a) Power generation, (b) volume gain ratio and (c) driving forces in single- and multi-stage PRO.

The project led to three peer-reviewed journal publications during this report period. Three undergraduate students presented their work at the 13th International Symposium on Process Systems Engineering and the paper was included in the conference proceedings. Two students joined Toyota and Andeavor as engineers and one went to graduate school. The PI received the Provost’s Award of Excellence in Scholarly and Creative Activities in 2018.