# Hydraulic fracture in anisotropic elastic material

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## **PI: Egor Dontsov**

Assistant Professor Department of Civil and Environmental Engineering Cullen College of Engineering, University of Houston 713-743-0848, edontsov@central.uh.edu

#### Abstract

This work considers the problem of a planar hydraulic fracture in a transversely isotropic (TI) material. In contrast to isotropic formations, in which a penny-shaped vertical fracture is produced, the elastic anisotropy changes the aspect ratio of the fracture. The latter aspect ratio of a uniformly pressurized (or "dry") elliptical crack can be estimated for given TI elastic parameters. We extend this result to hydraulically driven fractures by using the near-tip asymptotic solutions of the crack for different regimes of propagation. The crack aspect ratios for the toughness-dominated regime, the viscosity-dominated regime, and the regime determined by yield stress of Herschel-Bulkley fluid are calculated. The aspect ratio for the toughness-dominated regime is in agreement with the known solution for the "dry" crack, while the result differs for other regimes. The outcomes of this study enable one to quickly estimate the effect of elastic anisotropy on hydraulic fracture geometry for a given properties of the material.

## 1. ASPECT RATIO OF AN ELLIPTICAL FRACTURE IN ANISOTROPIC MATERIAL

Finely layered rock formations often obey anisotropic behavior globally. To construct a physically admissible transversely isotropic material, we first consider a periodically layered material with two types of isotropic layers. Each layer is characterized by Young's modulus  $E_i$ , Poisson's ratio  $\nu_i$ , and thicknesses  $h_i$ (i=1,2), as shown in Fig. 1. For simplicity, we assume that all layers have the same thickness and Poisson's ratio, i.e.  $h_1 = h_2$  and  $\nu_1 = \nu_2$ . The procedure of Backus averaging (Backus, 1962) is applied to determine elastic constants of the equivalent TI material: horizontal and vertical Young's moduli  $E_h, E_v$ , Poisson's ratios  $\nu_h, \nu_v$  and vertical shear modulus  $G_v$ .



Figure 1: Representation of a periodically layered formation by a homogeneous TI material via the concept of homogenization. Unit cell of the periodic material consists of two layers with thicknesses  $h_1$  and  $h_2$ , Young's moduli  $E_1, E_2$ , and Poisson's ratios  $\nu_1, \nu_2$ . The TI material has horizontal and vertical Young's moduli  $E_h, E_v$ , Poisson's ratios  $\nu_h, \nu_v$ , and vertical shear modulus  $G_v$ .

We consider the problem of a vertical hydraulic fracture in a TI material. Figure 2 shows the elliptical crack which lies in the xz-plane of the TI material with the axis of symmetry z. The aspect ratio of the ellipse is defined as  $\gamma = a/b$ . The aspect ratio of the "dry" or uniformly pressurized fracture was obtained in Laubie and Ulm, 2014b in terms of effective plane strain Young's moduli  $(E'_0, E'_{\pi/2})$  of the fracture tips which are determined by anisotropic elastic material properties and in turn depend on the ratio of the Young's moduli of the individual layers  $E_1/E_2$ . We have extended this result of [2] to a hydraulically driven fracture propagating in different limiting regimes. A general model of non-Newtonian fluid with the yield



Figure 2: Left: Vertical elliptical crack with the semi-axes a, b in a TI material. The crack opening is shown in the zy-plane with the maximum opening  $w_0$ . Right: Aspect ratio of an elliptical fracture versus the ratio of Young's moduli of periodically layered material  $E_1/E_2$ . Lines in the figure correspond to aspect ratios for different regimes of fracture propagation:  $\gamma_k^K$  - toughness-dominated regime with equal horizontal  $(K_{Ic}^h)$  and vertical  $(K_{Ic}^v)$  fracture toughnesses;  $\gamma_k^E$  - toughness-dominated regime with equal contributions from  $K_{Ic}^v$  and  $K_{Ic}^h$  to fracture energy;  $\gamma_{\tau}$  - fluid yield stress dominated regime;  $\gamma_{m,n=0.5}$  and  $\gamma_{m,n=1}$  viscosity-dominated regimes with different flow indices n.

stress (Herschel-Bulkley) is used. Fig. 2 shows the aspect ratio that is calculated for different regimes of propagation versus the ratio of Young's moduli,  $E_1/E_2$ , of the individual layers of layered material and for Poisson's ratio  $\nu = 0.2$  (the same for both layers) [3]. The aspect ratio of the elliptical crack increases with the ratio  $E_1/E_2$  which reflects the extent of anisotropy. The material anisotropy has the most significant influence on the aspect ratio of the crack for the toughness-dominated regime. For the viscosity-dominated and fluid yield stress dominated regimes, the influence of anisotropy is smaller. Aspect ratios for fluids with flow indices n=1 (Newtonian fluid) and n=0.5 are shown in Fig. 2. The flow index does not significantly affect the aspect ratio.

## 2. IMPACT OF THE RESEARCH

This work resulted in a conference paper [3] and a presentation for the graduate student, who primarily worked on this project. In addition, the graduate student was offered a summer internship to incorporate the results of this work into a commercial hydraulic fracturing simulator. Finally, a full journal paper that is based on this work is in preparation.

From the PI perspective, despite of heavy math involved, the results are very interesting and transparent. It is now possible to quantify whether elastic anisotropy is important or not for a given problem parameters. Solution of this problem, however, just scratches a surface of full understanding of elastic anisotropy. In particular, it is firstly important to validate these results with a fully coupled hydraulic fracturing simulator. Then, one may investigate what elastic parameters are the most influential for fracture crossing from one anisotropic layer to another.

## REFERENCES

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