Summary of Progress: Energy applications such as hydraulic fracturing require understanding of the mechanics of shear-dominated flows of dense suspensions. Specifically, mixtures of particles (termed “proppants”) in a viscous fluid are injected into newly created fractures to increase fracture conductivity and prevent closure of the fracture upon the cessation of flow. The proppant distribution is thus critical in such applications. Two-fluid models (TFMs) are becoming popular for complex multiphase flow applications, however, a general computational TFM framework, which can be used to study proppant transport, is lacking. The PI and his team developed a computational TFM for the flow of such dense non-Brownian suspensions. Our TFM is built upon the foundations of phase-averaged multifluid continuum mechanics and implemented using finite-volume methods on top of the OpenFOAM platform, branching its code base from the twoPhaseEulerFoam solver of Rusche, Passalacqua et al. The result is open-source computational tool that is available for anyone to use and modify. Our achievement allows accurate simulation of particle migration in dense slurries in complex three-dimensional (3D) geometries, as highlighted in Figure 1.

Figure 1: Direct numerical simulation of particle migration in the pressure-driven flow of a dense suspension through a single 3D element of the so-called staggered herringbone mixer. The flow (a) and concentration (b) fields were computed using our proposed two-fluid model (TFM) implemented in OpenFOAM. The final cross-sectional concentration profile, having started from a spatially uniform suspension state, agrees well with experiments.

This task builds upon our achievements in the first year of the project. Previously, we validated the TFM against published experimental data on particle migration. Therefore, the anisotropic stress model we have used for the particle phase is able to predict the suspension mechanics accurately. However, we make no assumptions on the inertial terms of the suspension (Reynolds number), allowing the TFM to capture a fuller range of suspension dynamics, such as those shown in Figure 1. The simulations were carried out on Purdue University’s Brown Computing Cluster, on which we purchased computational nodes through the generous support of this ACS PRF award. In doing so, a postdoctoral fellow trained a graduate (PhD) student and a summer visiting undergraduate student in the use of high-performance computing (HPC) resources. These skill are invaluable for modern science and engineering careers.

Ongoing Research Activity: Next, we set out to address the central hypothesis of the research project: Can the flow conduit’s geometry control particle migration? To this end, we considered a simple model of a shaped fracture, namely a variable-width Hele-Shaw cell, as shown in Figure 2. A power-law variation of the width, say $b(x) \propto (x/L)^n$ ($0 < n < 1$), is typical of hydraulic fractures. A dense suspension flowing towards this static fracture’s tip, or away from it, represents the injection and transport of proppants. Buoyancy is taken into account, as well as the viscosity of the suspending fluid, which can be lower (“slickwater”) or higher (polymeric mixtures) than the hydrocarbons present in the fracture. We are performing 3D direct numerical simulations using our proposed TFM framework, and supplementing the latter with simplified (but analytically tractable) reduced-order depth-averaged models.

Following along the lines of recent work by Saha, Salin and Talon, we derived a one-dimensional (1D) model of a fluid–fluid interface $h(x, t)$ separating the injected dense suspension from the hydrocarbons (see Figure 2). This interface is not a sharp surface-tension-laden boundary (as in our work on instabilities in the first year of this project)
but, rather, it is a region over which the particle concentration varies rapidly. In dimensionless variables, we obtained a variable-coefficient nonlinear diffusion equation:

$$\frac{\partial h^*}{\partial \tau} - \frac{1}{\xi^n} \frac{\partial}{\partial \xi} \left( \xi^n \Psi(h^*) \frac{\partial h^*}{\partial \xi} \right) = -1. \quad (1)$$

A jammed particle-dense layer can be left behind the spreading front, which is represented by the sink term on the right-hand side of Equation (1). As an example, we take the initial interface shape to be a sharp step-like function (bold curve in Figure 3(a)), and obtain its evolution by solving Equation (1) numerically. Of importance is that there exists a stoppage location, $\xi_s$, at which the current stops and reverses direction of propagation ($\xi_s \approx 0.21267$ in Figure 3(a)).

This effect has implications on the spread of proppants in fractures and the ability to well-distribute the materials. The existence of a stoppage location is due to the balance of the horizontal flux of material due to flow with the vertical flux of particles as they sediment into a jammed layer along the bottom of the fracture. Depending on the direction of propagation of the suspension (with or against the width gradient $db/dx$), the horizontal or the vertical flux may dominate. When the two fluxes are balanced, at some instant of time, the height of the suspension layer does not change, and the current stops at the stoppage location $\xi_s$. Figure 3(b) shows how this run-out distance (i.e., $\xi_s$) varies with the fracture shape (i.e., the exponent $n$ in $b(x)$) and the viscosity ratio $\mu_{BT}$ of the “bottom” fluid (displacing) to “top” fluid (displaced). In parallel, we are performing 3D direct numerical simulations to understand the structure of the suspension (nonuniform particle volume fraction in the cross-section), as it reaches the stoppage location.

Career Impact: This research is the dissertation topic of the PI’s first PhD student at Purdue, Daihui Lu, who is developing theoretical and numerical models of multiphase flows and instabilities in the presence of geometric variations. Dr. Federico Municchi, a postdoctoral scholar not funded by the award, collaborated with us on the OpenFOAM software development, and he gained mentorship experience. The ACS PRF award supported Daihui Lu’s presentations on this topic at the American Physical Society’s Division of Fluid Dynamics meetings in 2018 and 2019 (upcoming), affording her the major professional development opportunity of presenting at the fluid mechanics field’s most important scientific conference. In the second year of the project, two undergraduates gained research experience under this research project: Masashi Nishiguchi (Summer Research Fellow) and Ahmad Faraz Badar (S.N. Bose Fellow). They were mentored by Daihui Lu and the PI. Both of them are considering graduate studies in engineering.

![Figure 2: Schematic representation of a gravity-driven suspension current spreading in a thin fracture-like geometry (a Hele-Shaw cell). The fracture width $b(x)$ is allowed to be a function of the streamwise coordinate $x$. The particulate suspension is the denser fluid that flows along the bottom of the fracture.](image-url)

![Figure 3: Results from a reduced-order 1D model of a the spreading of a dense particular suspension in a shaped fracture. (a) Evolution of the interface between the invading particle-laden fluid and the defending clear fluid. (b) Dependence of the stoppage location on fracture geometry’s exponent $n$ and viscosity ratio $\mu_{BT}$ between fluids.](image-url)