PRF# 57734-ND9

A Rigorous Theoretical Framework for the Spreading of Liquid Droplets on Soft Solid Substrates

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Background

The spreading of liquids on soft solid substrates is a problem of fundamental interest that is also relevant to numerous applications including oil recovery. Experimental data published nearly two decades ago reveal that substrate deformability can significantly slow the spreading of liquid droplets. However, there is currently no rigorous theoretical framework that (i) is capable of explaining these data and (ii) can be generalized to other situations of interest. The objective of this proposal is to develop such a framework using lubrication theory and finite-element methods. The lubrication-theory-based approach will yield a simplified set of equations that permits rapid parametric studies. The finite-element approach will be used to test the limits of applicability of the former approach and to overcome its limitations. The results of the proposed work will bridge the gap between theory and experiment, and will lay the foundation for a powerful computational tool that can be used to explore more general situations involving droplet spreading on soft solid substrates. The resulting knowledge will advance the rational design of soft solid substrates for a broad range of applications.

The lubrication-theory-based approach will exploit the fact the droplets are often thin, enabling us to simplify the full governing equations to yield a set of evolution equations that are much easier to solve. Such an approach also has great potential to yield deep physical insight since analytical relationships are more easily extracted from the simplified equations. Our hypothesis is that such an approach will be able to describe the experimental data qualitatively, and perhaps even quantitatively.

Recognizing that the assumption of thin droplets used in the lubrication-theory-based approach may not always be valid, we also propose solving the full governing equations via the finite-element method. Such an effort will not only yield a theoretical tool that can be applied to more general situations of interest (e.g., droplets that are not thin), but will also provide insight into the conditions under which the simplified lubrication-theory-based approach is valid. Our hypothesis is that the finite-element approach will be able to overcome the limitations of the lubrication theory-based approach.

The proposed work is fundamental in nature because it involves detailed consideration of assumptions underlying a theoretical framework and examining why the framework might work best under a certain range of experimental conditions. The development of computational tools as part of the proposed research is an outgrowth of developing such a fundamentals-based framework, i.e., the tools need to be developed to bridge the gap between theory and experiment.

The proposed work is of relevance to the petroleum industry in two broad ways. First, soft solid substrates are often made of polymers, many of which are petroleum-based. If we understood better the fundamentals of droplet spreading on soft solid substrates, this would make it easier to design the substrates for specific applications (e.g., water recovery, heat transfer, coating and printing, microfluidics, mechanical property measurement of soft solids, studies of spreading of cellular aggregates to understand tumor biophysics). Second, soft solids in the form of gels are used in various stages of oil-recovery operations. Because liquids are also present and wetting plays a key role in understanding oil recovery, it is of fundamental interest to better understand the spreading of liquid droplets on soft solid substrates.

Progress and Impact

During the past reporting period, we have developed a mathematical model to examine the spreading of perfectly wetting and partially wetting droplets on soft solid substrates. The solid is assumed to behave like a linear viscoelastic material, and the dynamic contact line is described using a precursor-film/disjoining-pressure approach. Nonlinear evolution equations describing how the heights of the droplet-air interface and droplet-solid interface depend on space and time are derived using lubrication theory and then solved numerically with finite-difference methods. Parametric studies are conducted to investigate how the solid thickness, viscosity, shear modulus, and wettability influence droplet spreading. It is found that softer substrates speed up spreading for perfectly wetting droplets but slow down spreading for partially wetting droplets. For perfectly wetting droplets, the gap between the droplet-air and droplet-solid interface near the contact line is larger for softer solids. This allows more liquid to be transported from the droplet bulk toward the contact line and leads to faster spreading. For partially wetting droplets, the gap is smaller for softer solids, which hinders liquid flow from the droplet bulk toward the contact line and results in slower spreading. The results of this work allow us to disentangle the effects of surface wettability and solid deformability, and suggest new physical mechanisms underlying experimental observations of droplet spreading on soft solid substrates. A paper describing this work is currently being prepared for journal submission.

This work has taken my research program in a new direction (wetting on soft solid substrates) and inspired several new ideas that will form the basis of grant proposals to other funding agencies. It has also provided the PhD student carrying out this work the opportunity to move his thesis research in a direction that is especially interesting to him. During the next reporting period, we plan to (i) generalize our model to describe more complex situations (e.g., droplets on inclined surfaces), and (ii) develop finite-element simulations to test the validity of our lubrication-theory-based model.